

Congestion diffusion and decongestion strategy in networked traffic

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We study the information traffic in Barabási-Albert scale free networks wherein each node has finite queue length to store the packets. It is found that in the case of shortest path routing strategy the networks undergo a first order phase transition i.e., from a free flow state to full congestion state, with the increasing of the packet generation rate. We also incorporate random effect (namely random selection of a neighbor to deliver packets) as well as a control method (namely the packet-dropping strategy of the congested nodes after some delay time T) into the routing protocol to test the traffic capacity of the heterogeneous networks. It is shown that there exists optimal value of T for the networks to achieve the best handling ability, and the presence of appropriate random effect also attributes to the performance of the networks.

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I. INTRODUCTION

In the last few years, complex networked systems have attracted much attention, ranging from the areas of physics, sociology, biology, to technology and many others [1, 2, 3, 4, 5]. It has been well proved that the topological features of underlying interaction networks have great impacts on the final outcomes of the dynamics taking place on them [1, 2, 3, 4, 5]. For example, the scale free topology of a network results in a vanishing threshold of epidemic spreading on it with the increase of the network size [6], also gives rise to a robust behavior against random failures and fragile for aimed attack [7]; the networks with homogeneous degree distribution, small average path length, and small clustering coefficient are found to be more easily to retain synchronization [8].

Among all spreading and transport problems (epidemic, opinion, cascading, information, etc.) investigated in various kinds of complex networks [2, 3, 5, 9, 10, 11, 12], the information transport in communication networks such as the Internet may be of practical importance [13], since it plays a more and more important role in our daily life, e.g., data resources, the e-business, online games, and so may others. The ever-increasing number of users of the Internet, hence the need of tremendous amount of information transport make it necessary study how the topological properties of the underlying infrastructures influence the traffic flow [14]. In the studying of information traffic in complex networks, the following two problems are mostly focused on [15, 16, 17, 18, 19, 20, 21, 22]: Given the underlying network, what is the efficient routing protocol to optimize the packet delivery; and given a routing protocol, what type structure of the network is optimal, i.e., the performance of the structure or evaluation of the effect of structure on the traffic (optimality is defined as the minimization of the average packet arrival time and the maximization of the packet handling capacity of the network). In this paper, we would like to investigate in detail how the underlying topological structure affects the data traffic in complex heterogeneous networks.

All too often, the buffer size of the nodes are assumed to be infinite, i.e., all nodes can receive data packets as many as possible [15, 16, 17, 18, 19, 20, 21, 22]. However, due to the physical constraint, we know that all data-processing machines have a finite buffer (or queue length, denoted by L in the remaining parts for simplicity) to store data packets. It is reasonable and necessary to take into account this fact in the traffic study. So, in the present paper, we discard the infinite buffer assumption, but rather consider finite buffer of each node, and study how this restraint affect the data traffic. All of our results in the following parts are presented for $L = 5$. Other selection of value of L does not change the qualitative behavior of the results shown below. In addition we also incorporate random effect as well as a control method into the routing protocol to study the performance of the networks. It is shown that due to the finite storage of the nodes, a first order phase transition emerges in the information transportation process. The random effect and the control method in the routing protocol are also found to have a great influence on the handling capacity of the networks.

II. THE MODEL

It has been proposed that scale free network topology is a suitable candidate for the structure of the internet at AS level [23, 24]. For simplicity, we use the well-known Barabási-Albert (BA) scale-free network model [25] as the physical infrastructure on top of which a packet delivery process is taking place. The BA model contains two generic mechanisms of many real complex systems: growth and preferential attachment [1, 25], which can be constructed as follows. Starting from m_0 nodes, one node with m links is attached at each time step in such a way that the probability \prod_i of being connected to the existing node i is proportional to the degree k_i of that node, i.e., $\prod_i = k_i / \sum_j k_j$, where j runs over all existing nodes. In the present work, the total network size is fixed as $N = 1000$ and the parameters are set to be $m_0 = m = 3$ (hence the average connectivity of the network is $\langle k \rangle = 6$ [1]). The degree distribution of the generated BA network $P(k)$, which denotes the probability of a randomly selected node in the network having exactly degree k , follows a power

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law $P(k) \sim k^{-\gamma}$ with the exponent $\gamma = 3$ in the large degree limit.

Each node of the underlying infrastructure acts both roles of a host and a router at the same time. Due to the finite storage capacity of the nodes, we now define that a node is congested if its buffer is fully filled by packets. The packet transmission on the network, i.e., the packet generating process and the packet delivery process, is implemented by a discrete time parallel update algorithm. At each time step, the probability for node i to generate a packet is R if there are some free place in its buffer (i.e., available of its queue), otherwise no new packet is inserted. Once a packet is created, its destination is chosen uniformly at random among the other $N - 1$ nodes of the network. Each newly inserted packet is placed at the end of the queue of the node which generates it. After all the nodes have finished the packet generating process, they start to deliver the packets stored in their queue, which are composed of all packets that were sent to them by their neighbors in the previous steps and packets created by themselves (if any). For simplicity, and without loss of generality, we assume that all nodes have the same processing capacity of one data packet per time step. The first packet in the queue is then sent to a neighbor following some routing protocol.

Although in modelling communication networks like the Internet, the routing process following the shortest path (SP) from a given source to its destination is usually preferable, many previous studies have shown that some certain degree of stochasticity can to some extent enhance the traffic handling capacity of the underlying network [26, 27, 28, 29]. On the other hand, it has been found that for the internet almost 90% data packets are forwarded along their SP from source to destination, and the deviation of traffic from the SP is only about 10% [30]. Inspired by these two factors, we introduce in our model some randomness for packet delivery process, namely, we let data packets be delivered along their SP from source to destination with a probability $(1 - p)$ [31], and be sent to a randomly neighbor with probability p . Whenever a packet is forwarded along SP, if there exist several shortest paths, we just select randomly one of them to deliver the packet. For $p = 0.0$, we recover the SP delivery protocol. For other values of p greater than zero, we incorporate random effect into the packet delivery. To make a close correlation to the realistic scenario of data traffic, here we restrain our studies for the values of p between 0.0 and 0.2. Once a packet reaches its destination through the above routing protocol, it will be removed from the system.

We want to remark that the packet generation rate R in our model is different from that in the case of the infinite queue length [15, 16, 17, 18, 19, 20, 21, 22]. In fact, since those congested nodes (whose buffer are fully filled) temporarily can not generate any new data packets, the *effective* average packet generation rate of the whole network is equal or less than R . Nevertheless, we also denote average packet generation rate by R for convenience.

III. RESULTS AND DISCUSSION

According to the above algorithm for packet delivery, one can expect that for large values of R , the whole network would fall easily into a fully congested state due to the finite queue length of the nodes. In Fig. 1(a), we show four typical time evolution series of the average fraction of congested nodes in the BA scale free networks with total size $N = 1000$ and average connectivity 6. The different styled lines correspond to different packet generation rate $R = 0.003, 0.009, 0.012$ and 0.02 , respectively. From Fig. 1(a) one can see that for sufficient small R , e.g., $R = 0.003$, the networks sustain a free flow state (absence of congestion), while for large values of R , the networks are doomed to jam and the fraction of congested nodes in the networks increase very fast with the increment of time steps. The greater the value of R is, the more sharply the curves ascend. We have also implemented simulations for other values of N , $\langle k \rangle$ and L . The qualitative behaviors of the time evolution curves shown in Fig. 1 do not change (not shown here).

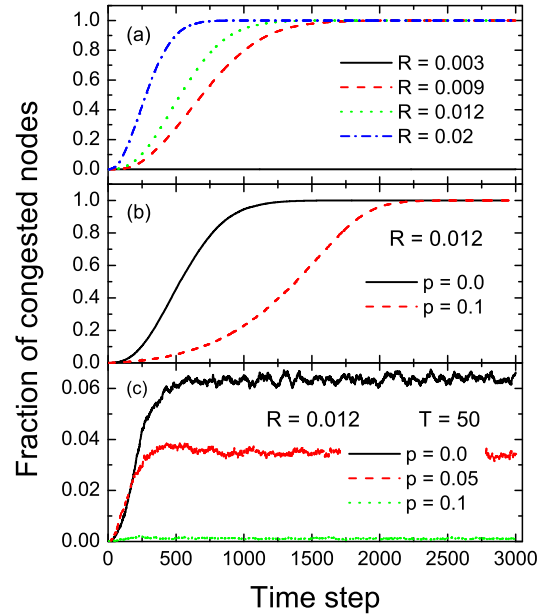


FIG. 1: Time evolution of the congested nodes in the BA scale free networks of size $N = 1000$ with $m = m_0 = 3$. (a) is for the data packets forwarded by using of SP routing protocol. (b) is for the case of some random effect involved in the packet delivery process. (c) is for the case of packet dropping strategy applied to the congested nodes. (see the text for details)

In Fig. 1(b), the time evolution of the average fraction of congested nodes is plotted for a special value of $R = 0.012$ with the incorporation of random effect in the routing protocols ($p = 0.1$). For comparison, the result for $p = 0.0$ is also shown. It is clear the curve for $p = 0.1$ ascends more slowly than that for $p = 0.0$, which indicates that some

appropriate degree of stochasticity for routing protocols can improve the handling ability of the network in accordance with previous observations in the case of infinite queue length [26, 27, 28, 29]. Despite of this point, we can see the network is still doomed to jam in the long time limit, which is also caused by the finite buffer size of the nodes. (Fig. 1(c) will be discussed later)

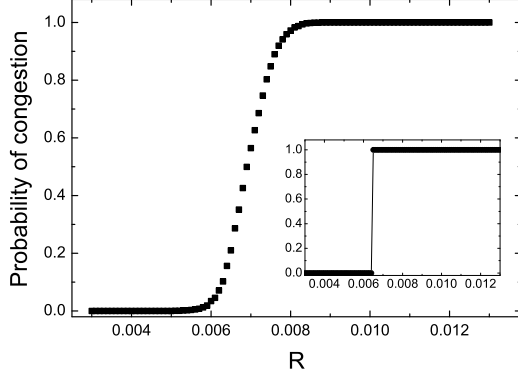


FIG. 2: Probability of congestion Vs. the data packet generation rate R in the BA networks of size $N = 1000$ with $m = m_0 = 3$. Each data point is obtained by averaging over 15000 samples (a set of 50 realizations of the BA network, and 300 independent experiments for each of them). The inset shows the result of a typical experiment of information traffic in a BA network. The Y-axis of the inset denotes the fraction of congested nodes.

Since whether the underlying networks are congested or not depends strongly on the packet generation rate R , we would first like to investigate the behavior of the congestion as a function of R . In the inset of Fig. 2, we show the result of a typical sample of the information traffic in a BA network. The packets are forwarded by using the SP routing protocol. It is obvious that a first order phase transition emerges at a certain value of R_c above which the network is doomed to congestion. Due to the finite size of the considered BA network ($N = 1000$ in the present case), the value of R_c for the traffic in different network realizations will vary to some extent. To obtain a more precise picture of the phase diagram, we have implemented 15000 samples including a set of 50 realizations of the BA network, and 300 independent experiments on each of them. We calculated the total number of times that the networks fall into full congestion by varying the value of R , and then the results are normalized by the total number of samples. The final results, i.e., the average probability of congestion as a function of R , are plotted in the main panel of Fig. 2, and the threshold R_c is about 0.005(5) in the present case (obviously, for other values of N , m_0 , L , this value will vary).

From Fig. 1 (a), we can see that, once some nodes are jammed, the congestion spreads very fast in the whole network if no control measure is deployed. From point view of developing control strategies to prevent the whole network from malfunction, it is valuable to figure out the detail knowl-

edge of the way that the congestion spreads through the network. Since the packets are delivered according to the SP protocol, the hubs (having large degrees and hence high betweenness) are more prone to congestion than those nodes with small degrees. A simple way of characterization of the congestion diffusion through the network is to study a convenient quantity, namely the average degree of congested nodes (if any), in numerical transporting experiments. We show in Fig. 3 this quantity for ten independent samples in the BA networks as a function of simulation time step. It is clear that the curves show an initial plateau whose values are far greater than the average connectivity of the network 6, which indicates undoubtedly that the large degree nodes in the network are preferentially congested. After the hubs are blocked, those nodes with largest degree in the remaining are successive to be congested, which is reflected by the smooth decreasing of the curves. Taken together, the dynamical spreading process of congestion is clear: with more and more packets inserted in the network, the hubs with high betweenness are firstly congested, and then the congestion spreads going towards those nodes with smaller and smaller degrees. Finally, the whole network is totally jammed, and the average degree of congested nodes is no other than the average connectivity of the BA networks.

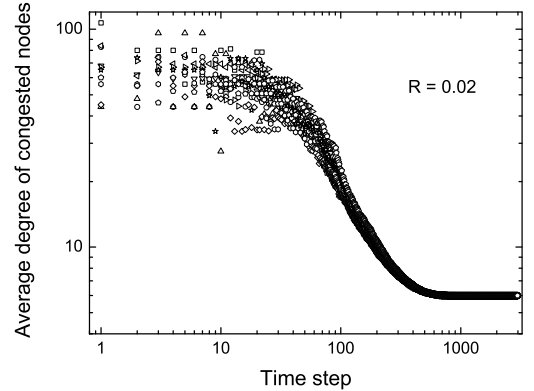


FIG. 3: Time behavior of the average degree of the congested nodes for congestion outbreaks ($R = 0.02 > R_c$) in BA networks of size $N = 1000$ with $m = m_0 = 3$. The shown results are for ten independent samples (plotted by different symbols).

From the above scenario of the congestion propagation in heterogeneous networks, one may realize that an efficient way to prevent the whole network from malfunction is to detect the blocked nodes in the very early stage of congestion, and to deal with the problem as soon as possible. However, due to the fluctuation of information flow in real communication systems [32] as well as many other physical restraints, it is inconvenient and even impossible to make immediate measurements and treatments on the congested nodes before a large scale breakdown of the system. More easier is to let the congested nodes themselves have the ability of getting out of jam

in terms of some appropriate instructions. As an alternative way, in the following part we implement a packet-dropping strategy for the congested nodes to prevent the whole network from jamming.

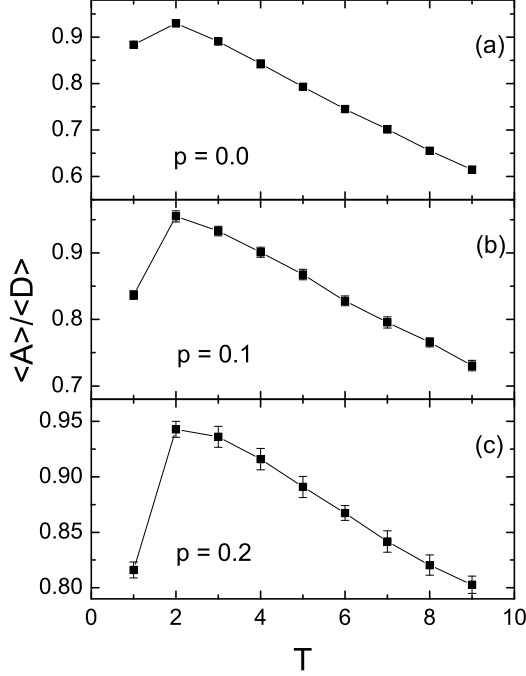


FIG. 4: The ratio of the average number of successfully arrived packets, $\langle A \rangle$, to the average number of dropped packets, $\langle D \rangle$, as a function of the delay time T for different values of p . The error bar denotes the standard deviation from a total of 30 traffic simulations on one realization of BA network. The parameter R is equals to 0.05.

We assume that if a node is congested and the persistence of this congested state exceeds some time T , then the node will simply empty its buffer to regain the capacity of receiving and handling packets. In practice, this method can be easily implemented by using control software, for example, the long time congested router can call for a pre-installed instruction to empty its buffer. In our computer simulations, if the congested state of a node lasts for more than T time step (for convenience, the time that a node is just congested is marked by $T = 1$), we reset its buffer size back to $L = 5$, and those stored packets in its buffer are discarded. Obviously, by doing so, we face the problem that many packets will be lost and disappeared forever in the network. On the other hand, however, if no strategy such as packet-dropping is implemented, the whole network would fall rapidly into collapse. On balance, an acceptable method is to select an appropriate parameter T to achieve as least packet-losing as possible, while keeping the whole network functional.

In Fig. 1(c), we show the time evolution of the fraction of congested nodes for $T = 50$ and several values of p . The

packet generation rate R is set to 0.012, which guarantees that the whole network is blocked without the implement of the strategy of packet-dropping [as displayed in Fig. 1(b)]. From Fig. 1(c), we note that even for so large value of T , the networks remain at low level congestion, and if some random effects are involved in the packet delivery, the congestion level may further decrease to a negligible level [the curve for $p = 0.1$ in Fig. 1(c)]. If T is too large, we know that the whole network is still easily to be jammed for large values of R ; if T is too small, however, there would be so many packets losing. Take both into consideration, there should be an intermediate value of T to achieve optimal performance. To evaluate the efficiency of the packet dropping method, we study the ratio of the average number of successfully arrived packets $\langle A \rangle$ to the average number of dropped packets $\langle D \rangle$ by tuning the values of the parameters T , p and R .

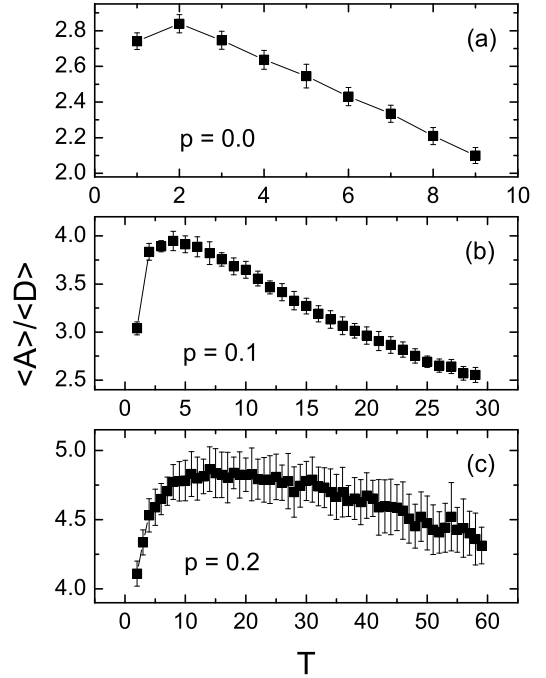


FIG. 5: As shown in Fig. 4, but for $R = 0.02$.

In Figs. 4-6 we show the quantity $\langle A \rangle / \langle D \rangle$, respectively, as a function of T for several values of p and R . The presented simulation results are obtained by averaging over 30 independent traffic realizations on the BA scale free networks of total size $N = 1000$ and $m = m_0 = 3$. For large packet generating rate $R = 0.05$ (Fig. 4), there exists an optimal value for $\langle A \rangle / \langle D \rangle$ at $T = 2$, which is insensitive to the detailed value of p . This result indicates that at times of high flux (large values of R), the most efficient way to alleviate traffic congestion and sustain the overall traffic handling ability of the heterogeneous networks is to empty the buffers of those congested nodes very immediately. However, it is wor-

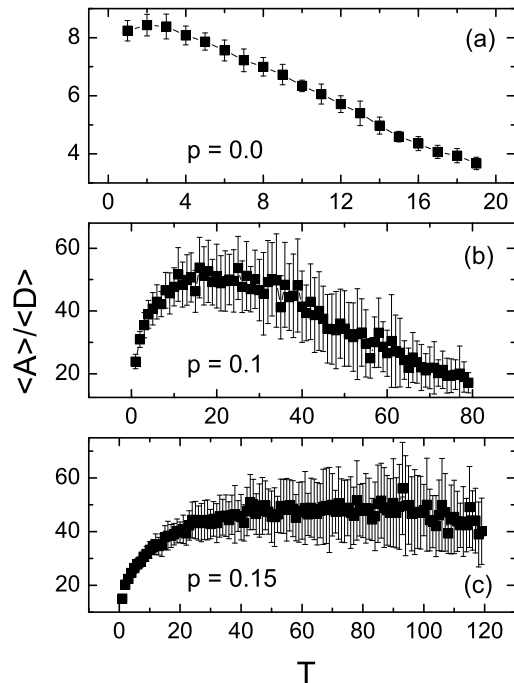


FIG. 6: As shown in Fig. 4, but for $R = 0.012$.

thy pointing out that the simulation results in Fig. 4 show clearly that the most appropriate time for the congested nodes to empty their buffer is not the time that they are just jammed, but one more time step after they are congested. This perhaps is due to the high rate of packets loss induced by immediate dropping of the packets stored in the congested nodes' buffer, which otherwise may be delivered in the next time step to their unblocked neighboring nodes.

The above picture, however, is changed for smaller (yet greater than the $R_c \approx 0.005$) values of R (Figs. 5 and 6). For $p = 0$, i.e., the packets are forwarded by using of the SP protocol, the most appropriate time for the congested nodes to empty their buffer is the same as before: at $T = 2$, namely one more time step after they are congested [Figs. 5(a) and 6(a)]. When there exist some stochastic effects in the packet delivery, i.e., for any value of p greater than 0, the most appropriate time for the congested nodes to empty their buffer shows non-trivial behavior, which depends closely both on the values of R and p . More precisely, the smaller the value of R as well as the larger the value of p , the larger the time T is suitable

[Figs. 5 (b) and (c), Figs. 6 (b) and (c)]. This means that at times of low flux, in order to achieve high packet arrival rate, the right way is just to let those congested node remain their blocked state for some appropriate time. It is worthy stressing that the condition of $p > 0$ should be satisfied, which is the actual case in realistic communication systems [30]. Finally, we want to remark that the curves shown in Figs. 5 and 6 are obtained from averages of 30 independent traffic experiments on one BA network realization. However, we have checked that for another independent realization of the underlying infrastructure, the shape of the curves may deviate to some extent, but all the qualitative properties of them (just as was shown in Figs. 5 and 6) remain absolutely unchanged.

IV. CONCLUSION

In summary, we have studied the information traffic in BA scale free heterogeneous networks. The nodes are endowed with finite buffer size and the same capacity of processing packets in each time step. It was found that for sufficiently small packet generation rate, the networks sustain a free flow state, while for large packet generation rate, the underlying infrastructures sooner or later fall into totally jammed. The phase transition for the heterogeneous networks going through the free flow state to the fully congested state with the increment of packet generation rate is of first order. Whenever the heterogeneous networks are going to be jammed, we have shown that the congestion takes first control of the hub nodes in the networks, and then it rapidly invades the whole network via a hierarchical cascade through progressively the nodes with smaller and smaller degrees. We have also investigated a control method, namely the packet-dropping strategy with a delay time T , to keep the whole network functional in the case of high packet generation rate. It was found that by using of this simple strategy, a network can sustain well its packet handling ability at the expense of lowering packet arrival rate. The efficiency and quality of the control method is determined by some correlated factors, e.g., the magnitude of the packet generation rate and the degree of stochasticity of the routing protocols as well.

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